Additive Equivalence During Turbulent Drag Reduction

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In a recent study of turbulent drag reduction (Virk and Wagger, 1990), we reported two examples of "additive equivalence" where different additives, following separate gross flow trajectories, nevertheless induced the same flow enhancements relative to solvent. The object of the present work is to provide new experimental information on additive equivalence, and to attempt its physical interpretation, to illuminate the role of the additive in drag reduction.

By way of background (Virk, 1975a; Hoyt, 1984), the drag-reduction phenomenon is bounded by two asymptotic, additive-insensitive flow regimes, of zero and maximum drag reduction, respectively, that envelop a third "polymeric" regime wherein additive properties exert an influence. Within the polymeric regime, two extremes of flow behavior, termed Types A and B (Virk, 1975b), are observed on Prandtl–Karman (abbr. P-K) coordinates of $1/f^{1/2}$ vs. log $Ref^{1/2}$, where Re is a diametral Reynolds number and f is Fanning's friction factor. In Type A drag reduction, exhibited by random-coiling macromolecules, a family of additive solutions yields a "fan" of friction factor segments that radiate outward from the vicinity of a common "onset" point on the P-K law, their slopes increasing with increasing concentration. Type A drag reduction thus increases both with increasing $Ref^{1/2}$ and with increasing c. In Type B drag reduction, exhibited by a variety of additives including extended polyelectrolytes and fibers, a family of additive solutions yields a "ladder" of segments that are roughly parallel to, but displaced upwards from, the P-K law, the more so with increasing concentration. Type B drag reduction is essentially independent of $Ref^{1/2}$, but increases with increasing c.

Both types A and B of drag reduction were induced in the present work by using saline solutions of the same polyelectrolytic additive, the initial conformation of which was altered from a collapsed random coil at high salinity to an extended chain at low salinity, while maintaining additive molecular weight and skeletal structure constant. This permitted sensitive experimental delineation of the influence of additive conformation upon drag reduction, uncluttered by changes in its other molecular attributes.

The experimental apparatus consisted of a test pipe, 10.9 mm ID × 2620 mm long, in a gravity-drained single-pass flow system that, with an entrance trigger, provided fully turbulent flow for solvent Re from 3,000 to 40,000. A partially hydrolyzed polyacrylamide (PAMH) additive called B1120 (Betz Laboratories, Trevose, PA) was used, of molecular weight $Mw \sim 20 \times 10^6$ and degree of backbone hydrolysis $\sim 50\%$. The additive was employed at concentrations c from 0.30 to 30 weight parts per million (wppm) in each of two saline solutions, respectively, 0.30 and 0.0003 N NaCl, wherein its intrinsic viscosities $[\eta]$ were ~ 3000 and 30,000 cm^3/g. The latter figures reflect the great differences in hydrodynamic coil-size between the collapsed and extended conformations of the additive. Pains were taken to minimize polymer degradation during solution makeup and to maintain identical polymer concentrations in solutions of different salinities. Experiments were run at a constant temperature $T = 25.1 \pm 0.1°\text{C}$. Flow rates and pressure drops were measured to within $\pm 1.0\%$ and $\pm 1.5\%$ of their absolute values, respectively. Flow-induced polymer degradation, which invariably accompanies turbulent drag reduction, was found to be negligible, even at the highest flow rates.

Gross flow results are presented in Figure 1, using P-K coordinates. The lines labelled L, N, and M, represent Poiseuille's law for laminar flow, the P-K law for Newtonian turbulent flow, and the maximum drag reduction asymptote (Virk, 1975a):

$$1/f^{1/2} = Ref^{1/2}/16 \tag{1}$$

$$1/f^{1/2} = 4.0 \log Ref^{1/2} - 0.4 \tag{2}$$

$$1/f^{1/2} = 19.0 \log Ref^{1/2} - 32.4. \tag{3}$$

In Figure 1a, solutions of B1120 in 0.30 N NaCl (collapsed conformation) exhibit the fan pattern characteristic of Type A behavior. At the lowest $Ref^{1/2}$, all solutions lie close to the solvent line N; the onset of drag reduction occurs at $Ref^{1/2*} \approx 300$; and for $Ref^{1/2} > Ref^{1/2*}$, the polymer solution trajectories fan outward from the onset point, their slopes exceeding the solvent slope by increments δ that increase roughly as the square root of polymer concentration. Both the onset wall shear stress, $\tau_w^* \approx 0.4$ Pa, and the specific slope increment, $(\delta/c^{1/2}) \approx 4$, derived from these data accord with the magnitudes expected from previous studies of Type A drag reduction by randomly coiled PAMH additives.

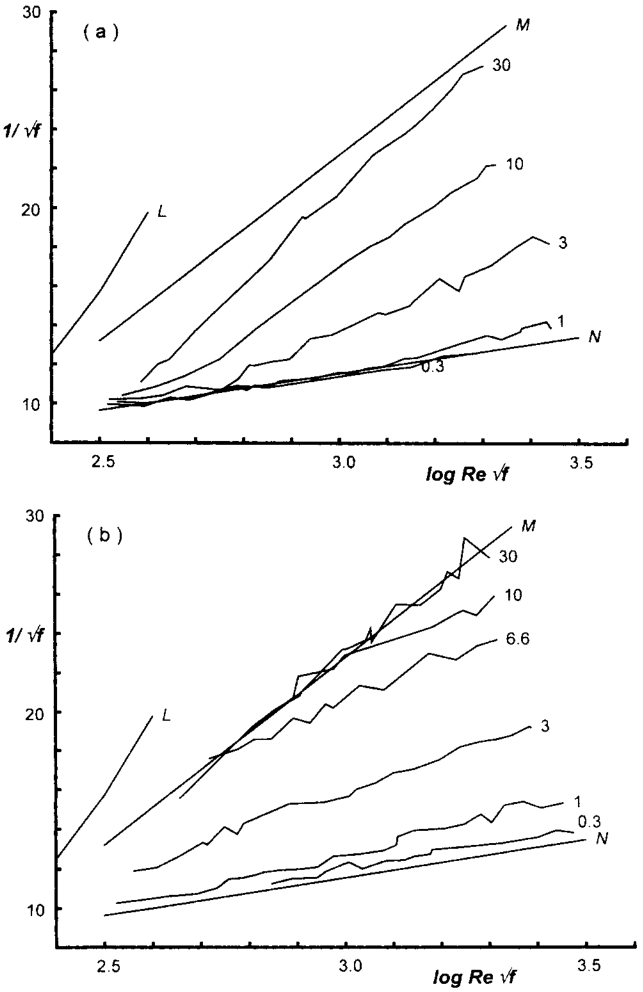

Figure 1. (a) Type A "fan" for collapsed conformation of B1120, in 0.3 N NaCl; (b) type B "ladder" for extended conformation of B1120, in 0.0003 N NaCl.

Lines L, N, M, respectively, correspond to Eqs. 1, 2, 3. Additive concentration, 0.3 to 30 wppm, is noted to the right of each trajectory.

In Figure 1b, solutions of B1120 in 0.0003 N NaCl (extended conformation) exhibit a "ladder" pattern characteristic of Type B drag reduction. These solutions reduce drag at the lowest turbulent $Ref^{1/2}$, so that no onset is visible. Indeed, for the cases $c = 6.6$ and 10 wppm the trajectories commence on the maximum drag-reduction asymptote M, and then exhibit "retro-onsets," at $Ref^{1/2\#} \approx 450$ and 650, into the envelope of lesser drag reduction, called the polymeric regime, that lies between M and N. For $c = 30$ wppm, the data all adhere to M, implying that retro-onset lay beyond the highest experimental $Ref^{1/2}$. Within the polymeric regime, all trajectories exhibit relatively shallow slopes, close to that of N. Thus, for fixed c, the extent of drag reduction increases only slowly with increasing $Ref^{1/2}$. However, at all $Ref^{1/2}$, drag reduction increases strongly with increasing c, giving rise to the distinctive ladder pattern.

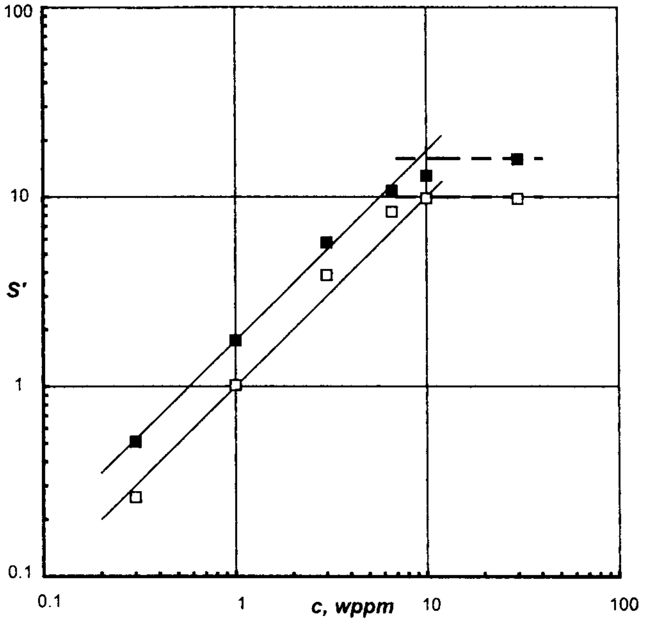

Figure 2. Effect of additive concentration on flow enhancement in Type B drag reduction.

Hollow and solid symbols depict data at $Ref^{1/2} = 650$ and 1600, respectively.

The effect of additive concentration in Type B drag reduction was deduced from the data in Figure 1b by using the "apparent slip," $S' = (1/f^{1/2}{}_p - 1/f^{1/2}{}_n)$ at fixed $Ref^{1/2}$, to measure the flow enhancement induced by a polymer solution (subscript p) relative to solvent (subscript n). Figure 2 is a log-log plot of S' vs. c at each of two fixed $Ref^{1/2}$, respectively, 650 and 1,600. In both cases, at low concentrations, S' increases linearly with increasing c over the 20-fold range $0.3 < c < 6.6$, as depicted by the solid lines of unit slope. At the highest concentrations, $c = 10$ and 30 wppm, S' becomes independent of c, attaining the maximum asymptotic values $S'_m \approx 10$ and 16 for the prevailing $Ref^{1/2}$, as indicated by the horizontal dashed lines. The linear portions of the S' vs. c relations provide a specific slip $[S'/c]$, characteristic of additive efficacy in Type B drag reduction. For the present B1120 at $Ref^{1/2} = 1600$, $[S'/c] \approx 1.8$ is of the same magnitude as the $[S'/c] \approx 4$ and 0.75 previously obtained at $Ref^{1/2} \approx 1500$ for the extended conformations of two other PAMH polymers, of $Mw \approx 23$ and 15×10^6, respectively (Virk, 1975a,b).

Comparisons between the Type A fan and the Type B ladder offer physical insights.

First, for a fixed total concentration, say $c = 10$ wppm, the trajectory for the collapsed conformation ($10A$) is always lower than that for the extended conformation ($10B$); thus, at any $Ref^{1/2}$ in the polymeric regime, the collapsed conformation yields less flow enhancement than the extended conformation, with $0 \le S'(10A)/S'(10B) \le 1$. Similar observations can be made at all concentrations in the 100-fold range, $0.3 < c < 30$. The universally greater efficacy of the extended conformation implies that it, more than the collapsed conformation, is the active additive state ultimately responsible for turbulent drag reduction.

Second, when trajectory $10A$ for $c = 10$ wppm of the collapsed conformation in Figure 1a is superimposed upon the data of Figure 1b, it intersects trajectory $3B$ for $c = 3$ wppm of the extended conformation at $(Ref^{1/2}, 1/f^{1/2}) \approx (650, 14.5)$, an "isoslip" point, where each of the two different solutions induce equal flow enhancements, $S'(10A) = S'(3B) \approx 3.7$. Formally, an isoslip point is the intersection of any two different gross flow trajectories in the polymeric regime; and at such a point the two additives associated with the intersecting trajectories are requivalent to one another in their ability to reduce turbulent drag. It is further interesting that at the $Ref^{1/2} \approx 650$ of the preceding isoslip point, where 10 wppm of the collapsed conformation was equivalent to 3 wppm of the extended conformation, the trajectory $10B$ for $c = 10$ wppm of the extended conformation exhibits flow enhancement $S'(10B) \approx 10$. Hence, we can construct a "slip triad" $S'[10A, 3B, 10B] \approx [3.7, 3.7, 10]$ comprising the respective flow enhancements of 10 wppm collapsed, 3 wppm extended, and 10 wppm extended additive, all at constant flow strength. In this triad, the ratio of collapsed-to-extended slips at constant concentration, $R_{SC} = S'(10A)/S'(10B) \approx 3.7/10 = 0.37$, is roughly the same as the ratio of extended-to-collapsed concentrations at constant slip, $R_{CS} = c(3B)/c(10A) \approx 3/10 = 0.30$. Several other isoslip points can be derived from the data in Figure 1a and 1b, and their corresponding slip triads all show approximate equalities between the ratio of collapsed-to-extended slips at constant concentrations R_{SC} and the ratio of extended-to-collapsed concentrations at constant slip R_{CS}. For example, at $Ref^{1/2} \approx 550$ the triad $S'[6.6A, 3B, 6.6B] \approx [3.4, 3.4, 8]$ gives $R_{SC} \approx 0.43$ and $R_{CS} \approx 0.45$, while at $Ref^{1/2} \approx 450$ the triad $S'[3A, 1B, 3B] \approx 0.9, 0.9, 3.2]$ provides $R_{SC} \approx 0.28$ close to $R_{CS} \approx 0.33$. The equality $R_{SC} \approx R_{CS}$ within a slip triad corresponding to an isoslip point suggests that, since the other molecular attributes of the additive were invariant, the ratio of collapsed to extended flow enhancements R_{SC} observed at fixed $Ref^{1/2}$ for given c likely represents the fraction of originally collapsed macromolecules that have become extended *in situ*, with only the extended macromolecules being effective in drag reduction.

Finally, the variation of R_{SC} with $Ref^{1/2}$ is of interest, as an indication of the turbulent flow strengths required to maintain that fraction of initially collapsed B1120 macromolecules in their extended states. Figure 3 is a semilog plot of R_{SC} vs. $Ref^{1/2}/Ref^{1/2*}$ for several concentrations from 1 to 10 wppm, the abscissa being normalized by the onset $Ref^{1/2*}$ at which the Type A trajectories commenced drag reduction. The observed slip ratios increase monotonically with increasing normalized flow strength according to the approximate relation:

$$R_{SC} = 1 - (Ref^{1/2}/Ref^{1/2*})^{-0.75};$$

$$(Ref^{1/2}/Ref^{1/2*}) > 1;\ 1 < c \text{ wppm} < 10. \quad (4)$$

Figure 3 and Eq. 4 show that in Type A drag reduction by the initially collapsed conformation of additive B1120, wall shear stresses respectively 2.1 ± 0.3, 6 ± 2, and 35 ± 10 times the onset value are required to achieve 0.25, 0.50 and 0.75 times the flow enhancement that would be exhibited in Type B drag reduction by the extended conformation of its macromolecular skeleton. These figures implicitly reflect the stretching of the macromolecule by the turbulent flow from random-coil toward full extension, and suggest that this process becomes progressively more difficult with increasing extension.

The preceding new data and physical interpretations extend our initial observation (Virk and Wagger, 1990) of additive equivalence. The variation of the ratio of collapsed to extended flow enhancement with flow strength has not previously been reported, and there appear to be no direct measurements of macromolecular extension during turbulent drag reduction, against which the present results might be compared. Interestingly, though, the data in Figure 3 bear a certain resemblance to the birefringence vs. strain-rate curves reported for the deformation of randomly coiled macromolecules in laminar extensional flows (Keller and Odell, 1985; Dunlap and Leal, 1987), and this possible analogy merits elaboration as a link between turbulent drag reduction and dilute polymer solution rheology.

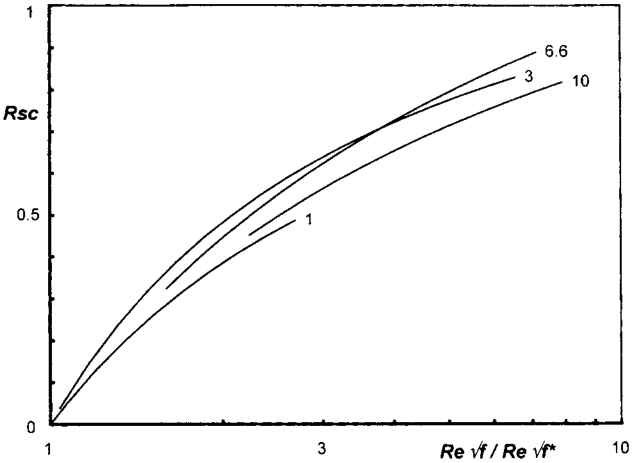

Figure 3. Dependence of the ratio of collapsed-to-extended flow enhancement on turbulent flow strength.

Additive concentration, 1 to 10 wppm, is noted to the right of each curve.

In summary, both Types A and B of turbulent drag reduction were induced by collapsed and extended conformations of a PAMH additive of $Mw \sim 20 \times 10^6$ in saline solutions. For a given additive concentration and $Ref^{1/2}$, flow enhancement by the extended conformation always exceeded that by the collapsed conformation. The ratio of collapsed-to-extended flow enhancements R_{SC} was physically identified with the fraction of originally collapsed macromolecules that had become extended *in situ*, with only the extended macromolecules effective in drag reduction. We estimate that about half of the initially collapsed additive macromolecules attain their extended conformations at a wall shear stress 6 ± 2 times that required for the onset of Type A drag reduction.

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